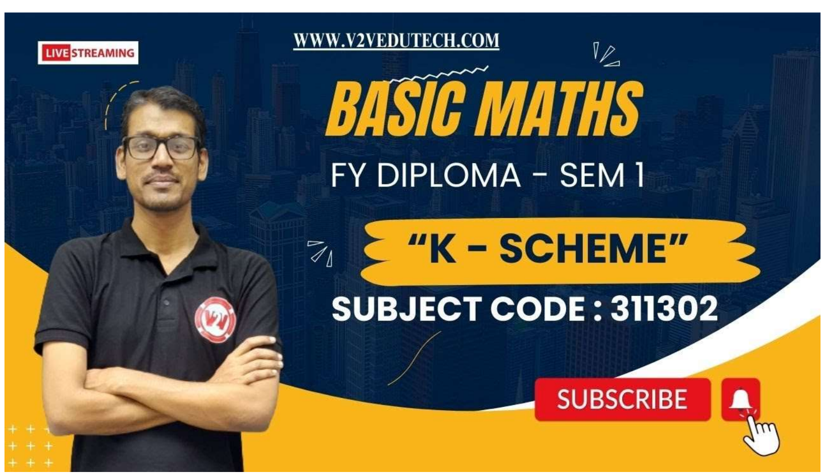
YouTube Lecture Links & Notes :
Application of Derivative: https://www.youtube.com/Application of Derivative

Join Whtassp Group
K – Scheme : https://chat.whatsapp.com/B5tS6rgj5pp4lRFHAWbc3P



Application of derivative

$\frac{d}{dx}\text{constant}=0$

$\frac{d}{dx}x^n=n\cdot x^{n-1}$

$\frac{d}{dx}kx^n=k\cdot\frac{d}{dx}x^n$

$\frac{d}{dx}a^x=a^x\cdot\log a$

$\frac{d}{dx}e^x=e^x$

$\frac{d}{dx}\log x=\frac{1}{x}$

$\frac{d}{dx}\sin x=\cos x$

$\frac{d}{dx}\cos x=-\sin x$

$\frac{d}{dx}\tan x=\sec^2 x$

$\frac{d}{dx}\cot x=-\operatorname{cosec}^2 x$

$\frac{d}{dx}\sec x=\sec x\cdot\tan x$

$\frac{d}{dx}\operatorname{cosec} x=-\operatorname{cosec}x\cdot\cot x$

(*) Product Rule

$$\frac{d}{dx}(u\cdot v)=u\cdot\frac{d}{dx}v+v\cdot\frac{d}{dx}u$$

(*) Division Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\cdot\frac{d}{dx}u-u\cdot\frac{d}{dx}v}{v^2}$$

(*) $\frac{d}{dx}(u\pm v\mp w)=\frac{d}{dx}u\pm\frac{d}{dx}v\mp\frac{d}{dx}w$

(*) Chain Rule

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$

$\frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$

$\frac{d}{dx}\operatorname{cosec}^{-1}x=\frac{-1}{|x|\sqrt{x^2-1}}$

$a^0=1$

$\frac{a}{0}=\infty$

$\frac{0}{a}=0$

$a\times 0=0$

$\frac{1}{x}=x^{-1}$

$\sqrt{x}=x^{1/2}$

$\sqrt[4]{x}=x^{1/4}$

$\frac{1}{\sqrt[6]{x}}=\frac{1}{x^{1/6}}=x^{-1/6}$

$a^m\times a^n=a^{m+n}$

$\frac{a^m}{a^n}=a^{m-n}$

$(a^m)^n=a^{m\times n}$

Application of Derivative

⊛ Finding eqⁿ of Tangent & Normal

$\pi = 180^\circ$

✳ Curve Equation & point (x_1, y_1)

$\hookrightarrow y = \_\_\_$

step ① find slope of Tangent

$$\left.\frac{dy}{dx}\right|_{at\ (x_1, y_1)} =$$

$$m_1 = \tan\theta$$

step ② finding eqⁿ of Tangent

1) Point (x_1, y_1)
2) Slope $= m_1$

$\therefore$ Eqⁿ of tangent

$$(y - y_1) = m_1(x - x_1)$$

step ③ Slope of Normal

$$\text{slope of Tangent} \times \text{slope of Normal} = -1$$

$$m_1 \times m_2 = -1$$

step ④ finding eqⁿ of Normal

1) Point (x_1, y_1)
2) Slope $= m_2$

Eqⁿ of Normal

$$(y - y_1) = m_2(x - x_1)$$

Q. Find eq^n of Tangent & Normal to curve

$$2x^2 - xy + 3y^2 = 16 \text{ at } (3,1)$$

$\Rightarrow$

Step ① finding slope of Tangent

$$2x^2 - xy + 3y^2 = 16$$

differentiating w.r.t x

$$\frac{d}{dx}\left[2x^2 - xy + 3y^2\right] = \frac{d}{dx}16$$

$$\frac{d}{dx}2x^2 - \frac{d}{dx}\underbrace{x}_{u} \times \underbrace{y}_{v} + \frac{d}{dx}3y^2 = 0$$

$$2\cdot\frac{d}{dx}x^2 - \left[u\cdot\frac{d}{dx}v + v\cdot\frac{d}{dx}u\right] + 3\frac{d}{dx}y^2 = 0$$

$$2(2x) - \left[x\cdot\frac{d}{dx}y + y\cdot\frac{d}{dx}x\right] + 3\frac{d}{dx}x^2 = 0$$

$$4x - \left[x\cdot\frac{dy}{dx} + y\cdot(1)\right] + 3(2x)\cdot\frac{d}{dx}x = 0$$

$$4x - x\cdot\frac{dy}{dx} - y + 6y\cdot\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 6y\cdot\frac{dy}{dx} = -4x + y$$

$$\frac{dy}{dx}(-x + 6y) = y - 4x$$

$$\frac{dy}{dx}(6y - x) = y - 4x$$

$$\frac{dy}{dx} = \frac{(y - 4x)}{(6y - x)}$$

$$\left.\frac{dy}{dx}\right|_{\substack{\text{at }(x,y)\\(3,1)}} = \frac{1 - 4\times 3}{6\times 1 - 3}$$

$$= \frac{-11}{3}$$

⊛ Slope of Tangent $= m_1 = -3.667$

Step ② finding eq^n of Tangent

1) Point $(x_1, y_1) = (3,1)$

2) Slope $= m_1 = -3.667$

Eq^n of Tangent,

$$(y - y_1) = m_1(x - x_1)$$

$$(y - 1) = (-3.667)(x - 3)$$

$$(y - 1) = (-3.667)x - 3\times(-3.667)$$

$$y - 1 = -3.667x + 11.001$$

$$Ax + By + C = 0$$

$$+3.667x + y - 1 - 11.001 = 0$$

$$3.667x + y - 12.001 = 0$$

$\hookrightarrow$ Eq^n of Tangent.

Step ③: Finding slope of Normal

$$\text{Slope of Tangent} \times \text{slope of Normal} = -1$$

$$m_1 \times m_2 = -1$$

$$(-3.667) \times m_2 = (-1)$$

$$\therefore m_2 = \frac{(-1)}{(-3.667)}$$

$$\therefore m_2 = 0.2727$$

Slope of Normal $= m_2 = 0.273$

Step ④: Finding eq^n of Normal

1) Point $(x_1, y_1) = (3,1)$

2) Slope $= m_2 = 0.273$

Eq^n of Normal

$$(y - y_1) = m_2(x - x_1)$$

$$(y - 1) = 0.273(x - 3)$$

$$y - 1 = 0.273x - 3\times 0.273$$

$$y - 1 = 0.273x - 0.819$$

$$Ax + By + C = 0$$

$$-0.273x + y - 1 + 0.819 = 0$$

$$-0.273x + y - 0.181 = 0$$

$\hookrightarrow$ Eq^n of Normal.

Q. Tangent is parallel to X-axis

$y = x^2 - 6x + 8$

⇒ step ① Finding slope of Tangent

$\because$ Tangent is parallel to X-axis

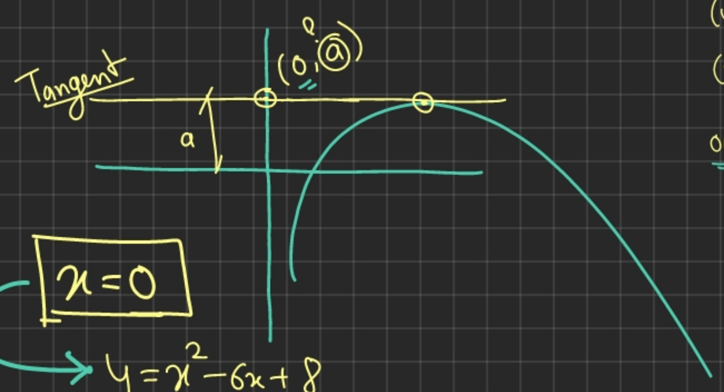
slope $= m_1 = 0$

$\because$ tangent is parallel to X-axis $\theta = 0^\circ$

$m_1 = \tan\theta$

$= \tan 0$

$m_1 = 0$



$\boxed{x = 0}$

$y = x^2 - 6x + 8$

$a = 0^2 - 6\times0 + 8$

$a = 8$

$\therefore$ Point $(0, 8)$

Q. Tangent is parallel to Y-axis

Step ①

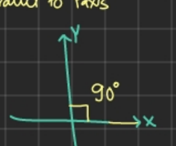
Tangent is parallel to Y-axis

$\theta = 90^\circ$

$m_1 = \tan 90$

$= \infty$

$m_1 = \frac{a}{0}$



step ② · Eqn of Tangent

$(y - y_1) = m_1(x - x_1)$

$(y - y_1) = \frac{a}{0}(x - x_1)$

$0(y - y_1) = a(x - x_1)$

$0 = a(x - x_1)$

$\frac{0}{a} = x - x_1$

$0 = x - x_1$

Q. Find eqn of Tangent & Normal to curve $y = 4xe^x$ at Origin. $(0, 0)$

step ① finding slope of Tangent

$y = 4xe^x$

diff. w.r.t x

$\frac{d}{dx} y = \frac{d}{dx} 4x \cdot e^x$

$= 4 \cdot \frac{d}{dx} x \cdot e^x$

$u \cdot v$

Q. Curve $4x^2 + 9y^2 = 40$ at $(1, 2)$

⇒ step ①: finding slope of Tangent

$4x^2 + 9y^2 = 40$

diff. w.r.t x

$\frac{d}{dx}(4x^2 + 9y^2) = \frac{d}{dx} 40$

$\frac{d}{dx} 4x^2 + \frac{d}{dx} 9y^2 = 0$

$4 \frac{d}{dx} x^2 + 9 \cdot \frac{d}{dx} y^2 = 0$

$4(2x) + 9 \frac{d}{dx} x^2 = 0$

$8x + 9 \cdot (2x) \cdot \frac{d}{dx} x = 0$

$8x + 18y \cdot \frac{dy}{dx} = 0$

$18y \frac{dy}{dx} = -8x$

$\frac{dy}{dx} = \frac{-8x}{18y}$

$\left.\frac{dy}{dx}\right|_{at (1,2)} = \frac{-8\times1}{18\times2}$

$= \frac{-8}{36}$

slope of Tangent $= m_1 = -0.222$

step ②: finding eqn of Tangent

1) slope $= m_1 = -0.222$

2) Point $(x_1, y_1) = (1, 2)$

Eqn of Tangent

$(y - y_1) = m_1(x - x_1)$

$(y - 2) = (-0.222)(x - 1)$

$y - 2 = -0.222x + 0.222$

$+0.222x + y - 2 - 0.222 = 0$

$0.222x + y - 2.222 = 0$

↳ eqn of Tangent

Step ③: finding slope of Normal

slope of Tangent $\times$ slope of Normal $= -1$

$-0.222 \times m_2 = -1$

$\therefore m_2 = \frac{-1}{-0.222} = 4.505$

step ④: finding eqn of Normal

1) Slope $= m_2 = 4.505$

2) Point $(x_1, y_1) = (1, 2)$

$\therefore$ Eqn of Normal

$(y - y_1) = m_2(x - x_1)$

$(y - 2) = 4.505(x - 1)$

$y - 2 = 4.505x - 4.505$

$-4.505x + y - 2 + 4.505 = 0$

$-4.505x + y + 2.505 = 0$

↳ eqn of Normal

Application of Derivative

Radius of Curvature

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$3/2$ → 1.5

Q. Curve eqn & point / angle

⇒ step ① find $\left.\frac{dy}{dx}\right|_{at\,(x_1,y_1)}$

step ② find $\left.\frac{d^2y}{dx^2}\right|_{at\,(x_1,y_1)}$

step ③ find Radius of Curvature

$$\rho = \frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Q. Find Radius of Curvature to curve $y = 2\sin x - \sin 2x$ at $x = \frac{\pi}{2}$

$\pi = 180°$

$\frac{\pi}{2} = \frac{180}{2} = 90°$

step ① finding $\left(\frac{dy}{dx}\right)$ at $x = \pi/2$

$y = 2\sin x - \sin 2x$

diff. w.r.t 'x'

$\frac{d}{dx} y = \frac{d}{dx} 2\sin x - \frac{d}{dx}\sin(2x)$

$= 2 \cdot \frac{d}{dx}\sin x - \frac{d}{dx}\sin x$

$= 2\cos x - \cos x \cdot \frac{d}{dx} x$

$= 2\cos x - \cos(2x) \cdot \frac{d}{dx} 2x$

$= 2\cos x - \cos(2x) \cdot 2 \cdot \frac{d}{dx} x$

$= 2\cos x - \boxed{\cos(2x)}\, 2$

$\frac{dy}{dx} = 2\cos x - 2\cdot\cos(2x)$

$\left.\frac{dy}{dx}\right|_{x=\pi/2} = 2\cos\left(\frac{\pi}{2}\right) - 2\cdot\cos\left(2\times\frac{\pi}{2}\right)$

$= 2\cos(90) - 2\cos(2\times 90)$

$= 0 - 2(-1)$

$\left.\frac{dy}{dx}\right|_{at\,x=\pi/2} = 2$ //

step ② finding $\left(\frac{d^2y}{dx^2}\right)$

$\frac{dy}{dx} = 2\cos x - 2\cdot\cos(2x)$

diff. w.r.t 'x'

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[2\cos x - 2\cdot\cos(2x)\right]$

$\frac{d^2y}{dx^2} = \frac{d}{dx} 2\cos x - \frac{d}{dx} 2\cos(2x)$

$= 2\cdot\frac{d}{dx}\cos x - 2\cdot\frac{d}{dx}\cos x$

$\frac{d^2y}{dx^2} = 2(-\sin x) - 2(-\sin x)\cdot\frac{d}{dx} x$

$= 2(-\sin x) - 2(-\sin 2x)\cdot\frac{d}{dx} 2x$

$= 2(-\sin x) - 2(-\sin 2x)\cdot 2\cdot\frac{d}{dx} x$

$\frac{d^2y}{dx^2} = 2(-\sin x) - 4(-\sin 2x)\cdot 1$

$\left.\frac{d^2y}{dx^2}\right|_{x=\pi/2} = 2(-\sin \pi/2) - 4\left[-\sin(2\times\pi/2)\right]$

$= 2(-1) - 4[0]$

$\left.\frac{d^2y}{dx^2}\right|_{at\,x=\pi/2} = -2$

Step ③ finding Radius of Curvature.

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$= \frac{\left[1+(2)^2\right]^{3/2}}{(-2)}$

$= \frac{\left[1+4\right]^{1.5}}{(-2)}$

$= \frac{5^{1.5}}{-2}$

$= \frac{11.18}{-2}$

Radius of Curvature $= -5.59$

$= 5.59$ units //

Application of Derivative

Maxima & Minima

step ① finding $\left(\frac{dy}{dx}\right)$

step ② find value of 'x' if $\left(\frac{dy}{dx}\right) = 0$

Step ③. finding $\left(\frac{d^2y}{dx^2}\right)$

Step ④ finding Maxima & Minima.

$\left.\frac{d^2y}{dx^2}\right|_{at\,x_1} = -ve$

$\therefore \frac{d^2y}{dx^2}$ is Negative

function is Maxima

OR

$\frac{d^2y}{dx^2} = +ve$

$\therefore \frac{d^2y}{dx^2}$ is positive

function is Minima.

Q. $y = x^3 - 18x^2 + 96x$

⇒ step ① : find $\left(\frac{dy}{dx}\right)$

$y = x^3 - 18x^2 + 96x$

differentiating w.r.t 'x'

$\frac{d}{dx} y = \frac{d}{dx} x^3 - \frac{d}{dx} 18x^2 + \frac{d}{dx} 96$

$\frac{dy}{dx} = 3x^2 - 18 \cdot \frac{d}{dx} x^2 + 0$

$\frac{dy}{dx} = 3x^2 - 18(2x)$

$\frac{dy}{dx} = 3x^2 - 36x$

step ②: find value of x if $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0 = 3x^2 - 36x$

$0 = 3x^2 - 36x$

$0 = x(3x - 36)$

$0 = a \times 0$

$0 = 0 \times a$

$\boxed{x = 0}$ $\qquad 3x - 36 = 0$

$x_1 = 0 \qquad 3x = 36$

$x = \frac{36}{3}$

$\boxed{x_2 = 12}$

991 MS

$0 = 3x^2 - 36x$

$0 = ax^2 + bx + c$

EQN 1

→ Degree

2 3

a? = 3

b? = -36

c? = 0

$x_1 = 0$

$x_2 = 12$ //

991 ES

3. $a_n x^2 + b_n x + c_n = 0$

$[\,3 \quad -36 \quad 0\,]$

$x_1 = 12$

$x_2 = 0$

Step ③ finding $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = 3x^2 - 36x$

diff. w.r.t 'x'

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx} 3x^2 - \frac{d}{dx} 36x$

$= 3\left(\frac{d}{dx} x^2\right) - 36\left(\frac{d}{dx} x\right)$

$= 3(2x) - 36(1)$

$\frac{d^2y}{dx^2} = 6x - 36$

step ④ finding Maxima & Minima.

$x_1 = 0$

$\left.\frac{d^2y}{dx^2}\right|_{x_1=0} = 6x - 36$

$= 6(0) - 36$

$= -36$

$\frac{d^2y}{dx^2}$ is Negative

∴ function is Maxima.

$y_{max} = x^3 - 18x^2 + 96x$

$= 0^3 - 0 + 0$

$= 0$ //

$x_2 = 12$

$\left.\frac{d^2y}{dx^2}\right|_{x_2=12} = 6x - 36$

$= 6(12) - 36$

$= 36$

∴ $\frac{d^2y}{dx^2}$ is positive

∴ function is Minima.

$y_{min} = x^3 - 18x^2 + 96$

$= 12^3 - 18 \times 12^2 + 96$

$y_{min} = -768$ //